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Representations of Solutions for Controlled Hybrid Systems

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An integral representation of solutions expressed by means of solutions of boundary-value problems of corresponding conjugate system is obtained for linear nonstationary controlled hybrid systems, This formula is very similar to the well-known Cauchy one in linear system theory. The results obtained are adjusted for the case of linear stationary hybrid systems with control.

Key words: controlled hybrid systems, difference-differential equations, nonstationary systems, piecewise continuous control, matrix function.

Character of hybrid systems

While studying real physical processes one can encounter both dynamic (differential) and algebraic (functional) dependences. Such processes are described by differential algebraic systems, with some equations being differential, the other — algebraic. These systems refer to the hybrid class. However, we have to admit that the term "hybrid systems" is overladen.

Generally speaking, hybridity implies inhomogeneous nature of the process considered or the methods for its investigation. The term "hybrid systems" refers to systems describing the processes or the objects with essentially different characteristics, e.g., those containing in the basic dynamics the continuous and discrete variables (signals), determinate and random magnitudes or actions, etc., with all these determining the character (nature) of hybrid systems.

There are numerous examples of hybrid systems. In the control field the following pattern of hybrid system is known: linear continuous time independent object described by linear differential equations with the mathematical model based on continuously operating recording device. The object is controlled by discrete linear time independent controller described by finite difference equations, discretely operating recording device being used. These types of systems are commonly studied in the levels called the discrete data systems or digital control systems.

Another standard example of hybrid control system is switching system where the behavior can be described by the finite number of dynamic models (systems of differential or difference equations) along with the set of rules for switching between the models.

One more field of hybrid systems represents studying qualitative properties, (e.g., stability) of dynamic systems described by difference-differential equations with discontinuous coefficients, systems with variable structure of dynamics.

In practice the classic example of hybrid system is heating and cooling systems in a dwelling house. A heater and an air conditioning along with the characteristics of heat flow form a controlled system. A thermostat is a discretely randomly controlled system which mainly deals with the symbols "too hot", "too cold" or "normal".

There are a lot of reasons for applying the hybrid models. First and foremost they include adequacy of these models, their valid simplification, the use of digital machines (control by computer programs); hybrid systems come into being while modeling the hierarchical structure of real control systems, specifically in describing dynamic, discrete, stochastic subsystems, complex systems etc.

Further information with respect to hybrid systems can be found in [1–10].

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In what follows we obtain for linear hybrid nonstationary systems the entire representation of solutions in the form of integrals based on solutions of the corresponding conjugate systems to generalize to these systems the known for ordinary systems representation by the formula of variation of constants (Cauchy formula) [11, 12]. Note, that the matters of solvability and representation of solutions of hybrid systems were considered before in the work [8] which analyzed mainly their "dynamic" part. It is worth noting that the paper in question suggests the new statement of initial problem.

Initial problem

We consider the simplest hybrid system

$$\begin{aligned} \dot{x}(t) &= A_{11}(t)\,x(t) + A_{12}(t)\,y(t) + B_1(t)\,u(t), \\ y(t) &= A_{21}(t)\,x(t) + A_{22}(t)\,y(t-h) + B_2(t)\,u(t),\ t \geq t_0. \end{aligned} \tag{1}$$

Here $x(t) \in R^n$, $u(t) \in R^r$, $y(t) \in R^m$; the components of matrix-functions $A_{11}(t) \in R^{n\times n}$, $A_{12}(t) \in R^{n\times m}$, $A_{21}(t) \in R^{m\times n}$, $A_{22}(t) \in R^{m\times m}$, $B_1(t) \in R^{n\times r}$, $B_2(t) \in R^{m\times r}$, $t \in R$, are piecewise continuous functions. In (1) at $t = t_0$ consideration was given to the right-sided derivative.

Initial conditions for the system (1) are given in the form

$$x(t_0+0) = x(t_0) = x_0,\ y(\tau) = \psi(\tau),\ \tau \in [t_0-h, t_0), \tag{2}$$

where $h = \text{const} > 0$, $x_0 \in R^n$, $\psi(\cdot)$ is a piecewise continuous in $[t_0-h, t_0]$ m-vector function.

We introduce the notation $T_t = \lim\limits_{\varepsilon\to+0}\left[\dfrac{t-t_0-\varepsilon}{h}\right]$ for $t > t_0$, $T_{t_0} = 0$.

Representation of solutions of nonstationary systems

Let the matrix-functions $X^*(t,\tau), Z^*(t,\tau), Y^*(t,\tau)$ are solutions of the conjugate system

$$\frac{\partial X^*(t,\tau)}{\partial\tau} + X^*(t,\tau)A_{11}(\tau) + Y^*(t,\tau)A_{21}(\tau) = 0,\ \tau \leq t,\ \tau \neq t-kh, \tag{3}$$

$$Y^*(t,\tau) = X^*(t,\tau)A_{12}(\tau) + Y^*(t,\tau+h)A_{22}(\tau+h),\ \tau \leq t, \tag{4}$$

$$Y^*(t,\tau) = 0,\ \tau > t, \tag{5}$$

$$X^*(t,t-kh-0) - X^*(t,t-kh+0) = Z^*(t,t-kh)\,A_{21}(t-kh), \tag{6}$$

$$Z^*(t,t-kh) = Z^*(t,t-kh+h)\,A_{22}(t-kh+h),\ k = 1,2,\ldots,T_t\,. \tag{7}$$

The relation (3) deals with the corresponding one-sided derivatives.

We denote by I_k the unity $k\times k$-matrix. The following theorem holds.

Theorem 1. The solution of system (1) with initial conditions (2) corresponding to the piecewise continuous control $u(\cdot)$ exists, is unique and can be calculated by formulae

$$X^*(t,t_0-0)\,x_0 + \int_{t_0-h}^{t_0} Y^*(t,\tau+h)\,A_{22}(\tau+h)\,\psi(\tau)\,d\tau +$$

$$+ \int_{t_0}^{t}(X^*(t,\tau)\,B_1(\tau) + Y^*(t,\tau)\,B_2(\tau))\,u(\tau)\,d\tau + Z^*(t,t-T_t h)\,A_{22}(t-T_t h)\psi(t-T_t h-h) +$$

$$+\sum_{k=0}^{T_t} Z^*(t,t-kh)B_2(t-kh)u(t-kh)=$$

$$=\begin{Bmatrix} x(t) \text{ at } t\ge t_0, \text{ if } X^*(t,t-0)=I_n\in R^{n\times n} \text{ and } Z^*(t,t)=0\in R^{n\times m} \\ y(t) \text{ at } t\ge t_0, \text{ if } X^*(t,t-0)=A_{21}(t)\in R^{m\times n} \text{ and } Z^*(t,t)=I_m\in R^{m\times m} \end{Bmatrix}. \tag{8}$$

Proof. That the solution of system (1) with initial conditions (2) and piecewise continuous control exists and is unique one can convince by integrating this system “by steps” [11, 12]. We prove the representation (8) by using the classic ideas of constructing conjugate boundary value problems [11–13].

By multiplying the first equation of the system (1) by piecewise continuous in the second argument matrix function $X^*(t,\tau)$ with the points of discontinuity of the first kind just at the time instants $\tau=t-kh, k=1,2,...,T_t$ and integrating by τ the range of t_0 and t we obtain

$$\int_{t_0}^{t}(X^*(t,\tau)\dot{x}(\tau)-X^*(t,\tau)A_{11}(\tau)x(\tau)-X^*(t,\tau)A_{12}(\tau)y(\tau)-X^*(t,\tau)B_1(\tau)u(\tau))d\tau=0. \tag{9}$$

By multiplying the second equation of the system (1) by piecewise continuous in τ matrix function $Y^*(t,\tau), \tau\in[t_0,t]$ and integrating by τ in the range of t_0 and t we obtain

$$\int_{t_0}^{t}Y^*(t,\tau)(y(\tau)-A_{21}(\tau)x(\tau)-A_{22}(\tau)y(\tau-h)-B_2(\tau)u(\tau))d\tau=0. \tag{10}$$

By assuming in the second equation of the system (1) the argument $\tau=t-kh, k=1,2,...,T_t$, multiplying the obtained equation by the matrix function $Z^*(t,t-kh)$ and summing over τ in the range of 0 and T_t we obtain

$$\begin{gathered}\sum_{k=0}^{T_t}Z^*(t,t-kh)y(t-kh)-\sum_{k=0}^{T_t}Z^*(t,t-kh)A_{21}(t-kh)x(t-kh)-\\ -\sum_{k=0}^{T_t}Z^*(t,t-kh)A_{22}(t-kh)y(t-kh-h)-\sum_{k=0}^{T_t}Z^*(t,t-kh)B_2(t-kh)u(t-kh)=0.\end{gathered} \tag{11}$$

We now add the equalities (9)–(11)

$$\begin{gathered}\int_{t_0}^{t}X^*(t,\tau)(\dot{x}(\tau)-A_{11}(\tau)x(\tau)-A_{12}(\tau)y(\tau)-B_1(\tau)u(\tau))d\tau+\\ +\int_{t_0}^{t}Y^*(t,\tau)(y(\tau)-A_{21}(\tau)x(\tau)-A_{22}(\tau)y(\tau-h)-B_2(\tau)u(\tau))d\tau+\\ +\sum_{k=0}^{T_t}Z^*(t,t-kh)y(t-kh)-\sum_{k=0}^{T_t}Z^*(t,t-kh)A_{21}(t-kh)x(t-kh)-\\ -\sum_{k=0}^{T_t}Z^*(t,t-kh)A_{22}(t-kh)y(t-kh-h)-\\ -\sum_{k=0}^{T_t}Z^*(t,t-kh)B_2(t-kh)u(t-kh)=0,\ t>t_0.\end{gathered} \tag{12}$$

Let us transform the terms in (12):

$$\sum_{k=0}^{T_t} Z^*(t,t-kh)\,y(t-kh) = \sum_{k=1}^{T_t} Z^*(t,t-kh)\,y(t-kh) + Z^*(t,t)\,y(t)\,, \tag{13}$$

$$\sum_{k=0}^{T_t} Z^*(t,t-kh)\,A_{21}(t-kh)x(t-kh) = \sum_{k=1}^{T_t} Z^*(t,t-kh)\,A_{21}(t-kh)x(t-kh) + Z^*(t,t)\,A_{21}(t)\,x(t), \tag{14}$$

$$\sum_{k=0}^{T_t} Z^*(t,t-kh)\,A_{22}(t-kh)y(t-kh-h) =$$

$$= \sum_{k=0}^{T_t-1} Z^*(t,t-kh)\,A_{22}(t-kh)\,y(t-kh-h) + Z^*(t,t-T_th)\,A_{22}(t-T_th)\psi(t-T_th-h) =$$

$$= \sum_{k=1}^{T_t} Z^*(t,t-kh+h)\,A_{22}(t-kh+h)\,y(t-kh) + Z^*(t,t-T_th)\,A_{22}(t-T_th)\psi(t-T_th-h), \tag{15}$$

$$\int_{t_0}^{t} Y^*(t,\tau)A_{22}(\tau)\,y(\tau-h)d\tau = \int_{t_0-h}^{t-h} Y^*(t,\tau+h)\,A_{22}(\tau+h)\,y(\tau)d\tau =$$

$$= \int_{t_0}^{t} Y^*(t,\tau+h)\,A_{22}(\tau+h)\,y(\tau)d\tau + \int_{t_0-h}^{t_0} Y^*(t,\tau+h)\,A_{22}(\tau+h)\,y(\tau)d\tau -$$

$$- \int_{t-h}^{t} Y^*(t,\tau+h)\,A_{22}(\tau+h)\,y(\tau)\,d\tau,$$

or by assuming $Y^*(t,\tau)=0$ for $\tau > t$ and taking into account the initial conditions (2) we obtain

$$\int_{t_0}^{t} Y^*(t,\tau)A_{22}(\tau)\,y(\tau-h)d\tau =$$
$$= \int_{t_0}^{t} Y^*(t,\tau+h)A_{22}(\tau+h)\,y(\tau)d\tau + \int_{t_0-h}^{t_0} Y^*(t,\tau+h)A_{22}(\tau+h)\psi(\tau)d\tau. \tag{16}$$

With the matrix-functions $X^*(t,\tau)$ having discontinuities of the first kind only at the points $\tau = t-kh,\ k=1,2,...,T_t$, integrating by parts in each interval $(t-kh,\, t-kh+h)$ and making use of integral additivity we obtain at $t-T_th > t_0$

$$\int_{t_0}^{t} X^*(t,\tau)\dot{x}(\tau)d\tau = \sum_{k=1}^{T_t} \int_{t-kh}^{t-kh+h} X^*(t,\tau)\dot{x}(\tau)d\tau + \int_{t_0}^{t-T_th} X^*(t,\tau)\dot{x}(\tau)d\tau =$$

$$= \sum_{k=1}^{T_t}\left(X^*(t,t-kh+h-0)\,x(t-kh+h) - X^*(t,t-kh+0)x(t-kh) - \int_{t-kh}^{t-kh+h} X^*(t,\tau)\,x(\tau)d\tau \right) +$$

$$+ X^*(t,t-T_th-0)\,x(t-T_th) - X^*(t,t_0+0)\,x_0 - \int_{t_0}^{t-T_th} X^*(t,\tau)\,x(\tau)\,d\tau =$$

$$= \sum_{k=1}^{T_t}(X^*(t,t-kh-0) - X^*(t,t-kh+0))\,x(t-kh) -$$

$$-\int_{t_0}^{t} \dot{X}^*(t,\tau)x(\tau)d\tau + X^*(t,t-0)x(t) - X^*(t,t_0+0)x_0. \tag{17}$$

In view of transformations (13)–(17) we rewrite the equality (12) in the form

$$-\int_{t_0}^{t} (\dot{X}^*(t,\tau) + X^*(t,\tau)A_{11}(\tau) + Y^*(t,\tau)A_{21}(\tau))x(\tau)d\tau -$$

$$-\int_{t_0}^{t} (X^*(t,\tau)A_{12}(\tau) - Y^*(t,\tau) + Y^*(t,\tau+h)A_{22}(\tau+h))y(\tau)d\tau -$$

$$-\int_{t_0-h}^{t_0} Y^*(t,\tau+h)A_{22}(\tau+h)\psi(\tau)d\tau - \int_{t_0}^{t} (X^*(t,\tau)B_1(\tau) + Y^*(t,\tau)B_2(\tau))u(\tau)d\tau +$$

$$+\sum_{k=1}^{T_t} (X^*(t,t-kh-0) - X^*(t,t-kh+0))x(t-kh) + X^*(t,t-0)x(t) - X^*(t,t_0+0)x_0 +$$

$$+\sum_{k=1}^{T_t} Z^*(t,t-kh)y(t-kh) + Z^*(t,t)y(t) -$$

$$-\sum_{k=1}^{T_t} Z^*(t,t-kh)A_{21}(t-kh)x(t-kh) - Z^*(t,t)A_{21}(t)x(t) - \tag{18}$$

$$-\sum_{k=1}^{T_t} Z^*(t,t-kh+h)A_{22}(t-kh+h)y(t-kh) - Z^*(t,t-T_th)A_{22}(t-T_th)\psi \times$$

$$\times (t-T_th-h) - \sum_{k=0}^{T_t} Z^*(t,t-kh)B_2(t-kh)u(t-kh) = 0, \quad t > t_0,\ t - T_th \neq t_0.$$

Since the matrix-functions $X^*(t,\tau), Z^*(t,\tau), Y^*(t,\tau)$ satisfy the conjugate system the relation (18) will take the form

$$-\int_{t_0-h}^{t_0} Y^*(t,\tau+h)A_{22}(\tau+h)\psi(\tau)d\tau - \int_{t_0}^{t} (X^*(t,\tau)B_1(\tau) + Y^*(t,\tau)B_2(\tau))u(\tau)d\tau +$$

$$+X^*(t,t-0)x(t) - X^*(t,t_0+0)x_0 + Z^*(t,t)y(t) - Z^*(t,t)A_{21}(t)x(t) - \tag{19}$$

$$-Z^*(t,t-T_th)A_{22}(t-T_th)\psi(t-T_th-h) - \sum_{k=0}^{T_t} Z^*(t,t-kh)B_2(t-kh)u(t-kh) = 0,$$

$$t > t_0,\ t - T_th \neq t_0.$$

By analyzing the previous transformations for $t - T_th = t_0$ we convince that the relation (19) is fulfilled by substituting the term $-X^*(t,t_0+0)x_0$ for $-X^*(t,t_0-0)x_0$. This is valid also for $t - T_t h \neq t_0$ since in the latter case $X^*(t,t_0+0) = X^*(t,t_0-0)$. Whence in view of the boundary conditions for the conjugate system we come to the relation (8) at $t > t_0$.

The direct verification illustrates that this relation remains valid also at $t = t_0$. Theorem 1 has been completely proved.

Corollary. The solution $x(t), y(t), t \geq t_0$ of system (1) corresponding to the initial conditions (2) and piecewise continuous control $u(\tau), \tau \in [t_0, t]$ exists, is unique and can be calculated by the formulae

$$x(t)=X^*(t,t_0)x_0+\int_{t_0-h}^{t_0} Y^*(t,\tau+h)A_{22}(\tau+h)\psi(\tau)d\tau+\int_{t_0}^{t}(X^*(t,\tau)B_1(\tau)+Y^*(t,\tau)B_2(\tau))\,u(\tau)d\tau,\ t\geq t_0, \tag{20}$$

where the initial conditions for the conjugate system are determined in the form

$$X^*(t,t-0)=I_n,\ Z^*(t,t)=0\in R^{n\times m},$$

and

$$y(t)=X^*(t,t_0-0)x_0+\int_{t_0-h}^{t_0} Y^*(t,\tau+h)A_{22}(\tau+h)\psi(\tau)d\tau+$$

$$+\int_{t_0}^{t}(X^*(t,\tau)B_1(\tau)+Y^*(t,\tau)B_2(\tau))u(\tau)d\tau+Z^*(t,t-T_th)A_{22}(t-T_th)\psi(t-T_th-h)+ \tag{21}$$

$$+\sum_{k=0}^{T_t} Z^*(t,t-kh)B_2(t-kh)u(t-kh),\ t\geq t_0.$$

Here the initial conditions of the conjugate system (3)–(7) are specified in the form $Z^*(t,t)=I_m$, $X^*(t,t-0)=A_{21}(t)\in R^{m\times n}$.

Indeed, in case of (20) as (6) implies the function $X^*(t,\tau)$ jumps vanish. So the function is considered to be continuous at $\tau<t$. Specifically, $X^*(t,t_0-0)=X^*(t,t_0+0)=X^*(t,t_0)$. In case of (21) the function $X^*(t,\tau)$ has jumps at the points $\tau=t-kh$. At that with $t-t_0$ being multiple of h the jump is performed also at the point t_0 to yield in view of (6) the value of $X^*(t,t_0-0)$ in the formula of (21). Thus, the corollary has been proved.

Note. The matrix-function $X^*(t,\tau), \tau\leq t$ in the conjugate system (3)–(7) at the discontinuity points $\tau=t-kh,\ k=1,2,\dots,T_t$ is supposed to be left-continuous to consider at these points in the equation (3) left-hand derivatives at $\tau\leq t$. Then in representations of solutions by formulae (8), (21) one can write $X^*(t,t_0)$ instead of $X^*(t,t_0-0)$.

Representation of solutions of stationary systems

Let us consider the hybrid system (1) with constant matrices

$$A_{11}(t)=A_{11}, A_{12}(t)=A_{12}, A_{21}(t)=A_{21}, A_{22}(t)=A_{22},$$

$$B_1(t)=B_1, B_2(t)=B_2, t_0=0. \tag{22}$$

The following theorem holds.

Theorem 2. The solution of the system (1), (2), (22) corresponding to the piecewise continuous control $u(\tau), \tau\geq 0$ exists, is unique and can be represented in the form

$$X(t)x_0+\int_{-h}^{0} Y(t-\tau-h)A_{22}\,\psi(\tau)d\tau+\int_0^t (X(t-\tau)B_1+Y(t-\tau)B_2)u(\tau)d\tau+$$

$$+Z(T_th)A_{22}\,\psi(t-T_th-h)+\sum_{k=0}^{T_t} Z(kh)B_2u(t-kh)=$$

$$=\begin{cases} x(t) \text{ at } t\geq 0, \text{ if } X(0)=I_n \in R^{n\times n} \text{ and } Z(0)=0\in R^{n\times m} \\ y(t) \text{ at } t\geq 0, \text{ if } X(0)=A_{21}\in R^{m\times n} \text{ and } Z(0)=I_m\in R^{m\times m} \end{cases}, \tag{23}$$

where the matrix-functions $X(\cdot), Z(\cdot), Y(\cdot)$ are the solutions of the conjugate system

$$-\frac{dX(t)}{dt}+X(t)A_{11}+Y(t)A_{21}=0,\ t\geq 0,\ t\neq kh, \tag{24}$$

$$Y(t)=X(t)A_{12}+Y(t-h)A_{22},\ t\geq 0, \tag{25}$$

$$Y(t)=0,\ t<0, \tag{26}$$

$$X(kh)-X(kh-0)=Z(kh)A_{21}, \tag{27}$$

$$Z(kh)=Z(kh-h)A_{22},\ k=1,2,...,T_t\,. \tag{28}$$

In (24) at the points $t=kh,\ k=0,1,...,T_t$ the discontinuity of the matrix-function $X(t)$ consideration is given to the right-hand derivative, the matrix-function being assumed right-continuous. Theorem 2 validity follows from Theorem 1 subject to the fact that in a stationary case the solution of conjugate system (24)–(28) is the matrix-functions

$$X^*(t,\tau)=X(t-\tau),\ Y^*(t,\tau)=Y(t-\tau),\ Z^*(t,\tau)=Z(t-\tau),$$

where $X(\cdot), Z(\cdot), Y(\cdot)$ is the solution of the system (24)–(28).

We now consider the simplest example:

$$\dot{x}(t)=0,\ t\geq 0,$$

$$y(t)=y(t-h),\ t\geq 0,$$

$$x(t)\in R^n,\ y(t)\in R^m,$$

$$x(+0)=x(0)=x_0,\ y(\tau)=\psi(\tau),\ \tau\in[-h,0).$$

The conjugate system (24)–(28) is of the form

$$\frac{dX(t)}{dt}=0, t\geq 0,\ t\neq kh,$$

$$Y(t)=Y(t-h), t\geq 0,$$

$$Y(t)=0,\ t<0,$$

$$X(kh)=X(kh-0),$$

$$Z(kh)=Z(kh-h),\ k=1,2,...,T_t\,.$$

By (23) we obtain the solution in the form $x(t)\equiv x_0, t\geq 0$ if $X(0)=I_n$ and $Z(0)=0$, then $X(t)\equiv I_n$, $Y(t)\equiv 0\ \forall t\geq 0$, $Z(kn)\equiv 0,\ k=1,...,T_t$; $y(t)=\psi(t-T_t h-h)$, $t\geq 0$, if $X(0)=A_{21}=0$ and $Z(0)=I_m$, then $X(t)\equiv 0$, $Y(t)\equiv 0\ \ \forall t\geq 0$, $Z(kh)\equiv I_m,\ k=1,...,T_t$.

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